Macroeconomics

Monetary Policy for a Resource-Rich Economy and the Zero Lower Bound

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Abstract

The paper studies monetary policy (MP) under a zero lower bound (ZLB) on the basis of a DSGE model. The economy is open and highly dependent on the terms of trade (TOT). Dynamics are driven by a TOT shock and an external interest rate shock. The change in the transmission of these external shocks in the presence of ZLB is analyzed. Unlike developed economies, for resource-rich countries the ZLB problem becomes relevant during economic expansion when external economic conditions improve. Positive external shocks lead to strengthening of the national currency, a decrease in inflation and a decrease in interest rate under inflation targeting MP. The paper shows that, with an inflation target of 4% and no persistence in interest rates, a positive TOT shock of 1 standard deviation is sufficient to drive the economy into a ZLB. At the same time, if the economy faces ZLB, the impact of the shocks is reduced, since there is an increase in real interest rates, which restrains an increase in household consumption and, accordingly, aggregate demand. Optimal MP rules under the ZLB are analyzed. When conducting MP, it turns out to be optimal to maintain high inertia in interest rates and be less responsive to changes in inflation, which minimizes the likelihood of binding ZLB. Contrary to optimal MP rules, the current MP of the Bank of Russia, along with the inflation target of 4%, excludes the possibility of reaching the ZLB, but is far from the optimal degree of response to changes in inflation. It is also found that, under the current MP, the likelihood of reaching the effective lower bound (ELB) is quite significant.

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Introduction

The zero lower bound (ZLB) of nominal interest rates is a natural constraint that monetary authority has to take into account. In developed economies, when negative shocks lead to a downturn, central banks usually lower nominal interest rates in order to stimulate economic activity. In the face of large negative shocks or a prolonged sequence of minor negative shocks, the nominal interest rate may reach its zero bound and remain there for a long time, thus limiting the ability of central banks to influence aggregate demand and stabilize the business cycle.

The ZLB problem was faced by Japan in the late 1990s, the US and, to a lesser extent, Europe during and after the 2008 global financial crisis. This problem has inspired numerous studies on economic policy measures that reduce the risks of reaching the ZLB. Both options to raise the inflation target and to modify monetary policy (MP) rules are being discussed.

For example, [Coibion et al., 2012] used a model calibrated with data from developed economies. They estimated that the optimal inflation target under rare and costly episodes of reaching the ZLB would not exceed 2% per year. On the other hand, [Williams, 2009] concluded that the 2% target level for the US economy was an insufficient buffer.

To influence aggregate demand in the ZLB environment, the regulator can pursue an economic policy by forming expectations [Fujiwara et al., 2015], conducting forward guidance [Detmers et al., 2021; Moessner, Rungcharoenkotkul, 2019] and choosing between discretionary and commitment policy. [Eggertsson, 2011] noted that, when an economy is at the ZLB, a trusted regulator finds it beneficial to promise to raise the inflation target after entering the positive interest rate area: this raises both expected future inflation and current inflation, leading to a drop in real interest rates and an increase in current output. It turns out to be profitable to break the promise after exiting the zero interest rate regime. A discretionary policy regulator with no credibility would not have access to such a policy. [Schmidt, 2013] showed that, when the regulator is credible and able to implement a policy under commitment, the optimal policy leads to fewer losses for the economy from reaching the ZLB than a discretionary policy. At the same time fiscal policy has little impact on changes in public welfare, as monetary policy itself significantly reduces losses from zero interest rates. If the regulator is only able to pursue a discretionary policy, then the losses from being at the ZLB are higher with fiscal policy helping effectively reduce these losses.

Also, the regulator can influence aggregate demand while being at the ZLB by quantitative easing [Woodford, 2016]. However, whatever new measures the regulator uses while being at the ZLB, the ZLB constrains monetary policy.

Monetary policy under the ZLB may differ between resource-rich and developed countries. Monetary policy is countercyclical in developed countries: when negative shocks lead to lower output and inflation, the regulator cuts the interest rate, supporting the economy. The regulator in an export-oriented country under an inflation targeting regime lowers the interest rate in response to a positive TOT shock in order to offset the declining inflation on the back of exchange rate appreciation. Low interest rate additionally accelerates output, which makes MP procyclical to output. For both developed and resource-rich countries, being at the ZLB leads to an additional drop in output due to the limited effect of the MP. However, in developed countries being at the ZLB increases the volatility of output, since an additional drop in output occurs during a recession, while in resource-rich countries the ZLB reduces volatility, since an additional drop occurs during a boom. Therefore, the recipes for regulators in export-oriented countries may differ qualitatively and quantitatively from those in other countries. In this article, the quantitative differences between the MP for resource-rich countries and that for developed countries are not considered. However, it is further considered a resource-rich economy, and economic dynamics are the result of external shocks, so it is possible to say that the results obtained are typical for resource-rich countries.

Historical episodes of an exporting economy being at the ZLB are rare. An example would be Chile 2010 [Céspedes et al., 2014]. The episode is characterized by low inflation and by a drop in interest rates to almost zero during the copper price rally. At the same time, commodity prices have been quite volatile in recent decades. Understanding the reasons why resource-rich economies rarely meet the ZLB amid high commodity price volatility is one of the motivations for this work.

Trend inflation in Russia, which is a resource-rich country, has been declining over the past two decades, reaching and even falling below 4%, which is in line with the Bank of Russia's inflation target. This has increased the likelihood of the Russian economy reaching the ZLB. Given this, it would be relevant to analyze how the increased probability of reaching the ZLB could be reflected in the Bank of Russia's MP. The relevance of this study is also related to the fact that the Russian economy may face the ineffectiveness of monetary policy earlier than at zero interest rates (Section 6), which is known as the effective lower bound (ELB) problem.

In the paper the authors have constructed a dynamic stochastic general equilibrium (DSGE) model for a small open export-oriented economy (Appendixes 1–7) to study the ZLB problem for resource-rich countries. The model is calibrated in a manner typical for export-oriented economies. A number of parameters and shocks were estimated using Russian data (Section 2). The negative impact of the ZLB on an export-oriented economy is demonstrated with the use of impulse response functions (Section 3). The monetary authority's objective is presented in Section 4. The optimal parameters of the Taylor rule are presented and discussed in Section 5. In Section 6 the likelihood of the Russian economy to hit the ZLB and ELB is discussed.

1. DSGE Model and the Zero Lower Bound

To investigate the MP, a fairly standard DSGE model for export-oriented economies is constructed. The model includes a description of non-commodity (non-traded) and commodity (traded) goods production. Manufacturers use labor and capital to produce both types of goods. A special production factor "land" is used as a third factor to produce traded goods. Land reflects lower labor and capital costs in the exporting sector. "Land" rent is paid to households. The proceeds from commodity exports first go to the producer and are then spent on paying for the three production factors: the rents on capital, labor and "land" rent. Non-tradable goods and imported goods are combined into final goods that are spent on consumption and investment.

As is standard for DSGE models, households consume the product, offer labor to manufacturers, save money in the form of foreign bonds, and decide on the amount of capital to lease to manufacturers.

The model implements a New Keynesian approach: some markets are assumed to function inefficiently. Inefficiency mechanisms include non-tradable goods price rigidity, wages rigidity, consumption habits, investment adjustment costs, labor adjustment costs in the exporting sector, and foreign bonds adjustment costs.

The monetary policy authority follows the Taylor rule:

$$R_t^l - R^{l,ss} = \rho_r (R_{t-1}^l - R^{l,ss}) + (1 - \rho_r) \times \\ \times (\rho_{inf}(\pi_t - \pi^{ss}) + \rho_y (GDP_t / GDP_t^{pot} - 1)), \quad (1)$$

where R_t^l is the nominal interest rate, π_t is inflation, GDP_t is the total output of the exporting and domestic sectors, $R^{l,ss}$, π^{ss} are the values of interest rate and inflation in the long-term equilibrium, and GDP_t^{pot} is potential output, i.e. output in the economy with flexible prices and wages.

A more detailed description of the model is provided in Appendixes 1–7. Note that this model differs from [Andreyev, Polbin, 2019] in the absence of an entrepreneur responsible for implementing a financial accelerator mechanism. The presence of an entrepreneur does not bring anything important to this study and is not necessary.

In order to account for the ZLB, it is assumed that there is an additional constraint in the model

$$R_t^l \geq 0. \tag{2}$$

This constraint means that at any time period for any event either the relation and the inequality (2) apply strictly, or

$$R_t^l = 0. \tag{3}$$

In the case of (3), inflation is said not to be targeted by the monetary policy authority but is determined by free market forces.

A model where inequality (2) occasionally binds is non-linear. There is no single recognized method for solving stochastic models with inequalities. However, these methods have been actively developed in recent years (e.g. [Binning, Maih, 2017; Holden, 2016; Lepetyuk et al., 2020]). Hereinafter, the OccBin toolkit developed by [Guerrieri, Iacoviello, 2015] is used to find solutions to dynamic stochastic models with occasionally binding constraints. This tool searches for piecewise linear solutions to dynamic stochastic models with inequalities and uses MATLAB and Dynare. [Lepetyuk et al., 2020] note that the OccBin toolkit and some other methods provide very similar solutions.

OccBin uses a piecewise linear approximation of the model. To solve the model, it is necessary to determine the reference regime, in which, according to the method, the economy will return after some time and remain there forever in the absence of shocks. In this model the reference regime corresponds to the Taylor rule (1). It is also necessary to identify an alternative regime in which the economy may temporarily fall as a result of the shocks. In this model the alternative regime corresponds to the zero rate equation (3). Each regime is characterized by its own long-term equilibrium and by its own linearized system. The method assumes that after a shock the system will eventually return to an equilibrium state corresponding to the reference regime. The algorithm makes an assumption about what periods of time the system is in the alternative regime. The algorithm makes an attempt to calculate the equilibrium. If successful, the algorithm stops; if that fails, then the algorithm makes another assumption about the time periods with the alternative regime.

The DSGE model is described in more detail in Appendixes 1–7.

2. Economic Shocks and Calibration

Terms of trade shocks and foreign borrowing interest rate shocks are usually considered the key determinants of the business cycle for resource-rich economies. In this study it is decided to focus on these two shocks and leave the other types of shocks for future research.

A terms of trade shock ε_t^{TOT} determines the dynamics in the terms of trade $\tilde{P}_t^{res}$ in line with the AR(1) process:

$$\ln(\tilde{P}_t^{res}) = \rho_{res} \ln(\tilde{P}_{t-1}^{res}) + \varepsilon_t^{TOT}. \tag{4}$$

A foreign interest rate shock ε_t^{prem} sets the dynamics for the foreign interest rate R_t^f according to the equation:

$$R_t^f = \rho_{prem} R_{t-1}^f + (1 - \rho_{prem}) R^{f,ss} + \varepsilon_t^{prem}. \tag{5}$$

Estimating the standard deviations and autocorrelations of these shocks is an important stage of the study, as these values affect the probability of the economy to hit the ZLB. Therefore, the authors estimated the autocorrelation of the logarithm of the TOT ρ_{res}, the variance of the terms of trade shock ε_t^{TOT} (4), the autocorrelation of the foreign interest rate ρ_{prem}, and the variance of the foreign interest rate shock ε_t^{prem} (5) using historical data. The authors chose Russia as the representative country. Since oil, petroleum products, gas, and liquefied natural gas comprised an average of 60% of Russian merchandise exports from 2000 to 2019, the authors chose the real oil price as the terms of trade. Using real Brent crude oil price quarter data for the period from 1995 to 2019, the authors estimated ρ_{res} using OLS at 0.956, and the standard deviation ε_t^{TOT} at 0.144. Foreign borrowing rate R_t^f was estimated in a manner similar to [García-Cicco, García-Schmidt, 2020; Huidrom et al., 2020] as the sum of the three-year US Treasury bond yield and the Russian bond yield spread as calculated by JP Morgan.[1] The choice of the period used for the foreign interest rate is an issue in the literature. For instance, the standard deviation of the shock ε_t^{prem} is 1.1% for the period starting from 1998 and including the event of default on government bonds, and 0.17% for the period starting from 2003. Since a default is unlikely under current economic conditions, although it cannot be completely ruled out, the authors set the standard deviation ε_t^{prem} at 0.34%. The parameter ρ_{prem} is estimated at 0.84.

The authors calibrated structural parameters in a standard manner for DSGE models for export-oriented economies. It is assumed that the time preference coefficient $\beta = 0.995$. Capital depreciation rates for both

[1] JP Morgan's EMBI JPSSEMRU Index.

producing sectors were considered to equal 0.02 in the line with [Motto et al., 2010]. This corresponds to an annual depreciation of capital of 8%. Standard parameters for the elasticity of the production function for the non-traded sector were chosen (e.g. [Bernanke et al., 1999]): $\alpha_d = 0.35$, $1 - \alpha_d = 0.65$. The share of "land" factor costs in the output of traded goods was set at 25%[2], and the costs of the remaining factors (capital and labor) were distributed in the ratio of 0.35/0.65. Since the ratio of exports to GDP varies over a wide range from 0.078 to 0.64 for exporting economies [Benkhodja, 2014], the authors took the average value $Y_t^{ex}/GDP_t = 0{,}25$ which corresponds to the Russian economy. Imports were considered equal to exports in the long-term equilibrium. The value for labor adjustment costs in the exporting sector w^{ex} was taken approximately at the level of [Ambler et al., 2012; Dib, 2003]. Other rigidity parameters were taken from [Andreyev, Polbin, 2019]).

The authors chose long-run inflation value based on the inflation targets of export-oriented countries. Statistics show[3] that the annual inflation targets for many resource-rich countries (e.g. Chile, Mexico, Norway, and Russia) lie in the range of 2% to 4% p.a., and that the value of 4% p.a. lies within the range of many central bank targets. In this regard, the authors have chosen two divergent values of long-term inflation for the study: 0% p.a. as the minimum possible target and 4% as a representative target near the upper limit for resource-rich countries. By setting the long-term real annual interest rate at 2%, the authors obtained two long-term nominal interest rate values for this study: 2% and 6% p.a. It is worth noting that the nominal interest rate calibration along with the shock magnitude calibration are among the key factors behind the probability of the economy reaching the zero bound of nominal interest rates.

To make sure that the model calibration is adequate, the authors tested how well the model is able to describe the crisis of 2014–2015. To set up this experiment, the authors need to determine the actual monetary policy. For this purpose, the authors estimated a simple Taylor rule with the nominal interest rate as a function of the interest rate lag, current inflation, and the current indicator of economic activity. The Taylor rule was estimated for the period from 2010 to 2019. The MIACR rate on overnight interbank loans was chosen as the interest

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[2] If the authors consider the "land" production factor cost as an additional tax levied on the exporting sector, the chosen normalization of 25% corresponds to the difference between the tax burden of 55% on the exporting sector and 30% on the domestic sector, which is comparable to the Russian economy.

[3] http://www.centralbanknews.info/p/inflation-targets.html.

rate, while the growth rate of the consumer price index was chosen as the inflation rate. The authors used two alternative measures as the indicator of business activity: the deviation of real GDP from the potential level obtained using the Hodrick-Prescott filter and the growth rate of real GDP. In the regression under consideration, both indicators of economic activity turned out to be statistically insignificant. Therefore, the authors have settled on a simple specification where the interest rate depends on the interest rate lag and current inflation. The regression equation was estimated using the OLS method, which may produce biased estimates if monetary policy shocks affect inflation at the same point in time. However, the authors abstract from this potential problem, as the experiment under consideration is approximate, and an accurate assessment of the monetary policy parameters for the Bank of Russia is not the purpose of this paper. The parameter estimates are shown in Table 1. According to the obtained estimates, the coefficients for the dependence of the current interest rate on its lag and the degree of response to inflation are equal to $\rho_r = 0.902$ and $\rho_{inf} = 1.49$.

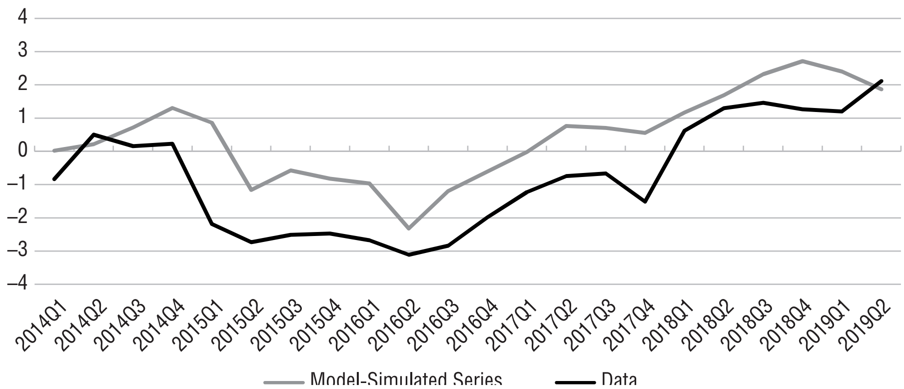
Table 1

OLS Estimation of Taylor Rule Equation for Russian Data from 2010 to 2019

Variable	Value	Standard Deviation	t-Statistic	p-Value
Constant	–0.0004	0.001	–0.330	0.743
MIACR Interest Rate, Quarterly, 1 Quarter Lag	0.902	0.043	20.899	0.000
Inflation, QoQ, Seasonally Adjusted	1.490	0.260	5.586	0.000
Observation Period	2010Q2-2019Q4			
Number of Observations	39			
R^2	0.951			
F-Statistic	225.724			
Probability (F-Statistic)	0.000			

Overall, there are many examples of a successful econometric estimation of DSGE model parameters for the Russian economy and filtering unobserved shocks (for example, [Ivashchenko, 2013; Kreptsev, Seleznev, 2018; Malakhovskaya, 2016; Shulgin, 2014]). In this paper, the authors rely on calibration of the model parameters, which is also common practice (for example, [Andreyev, Polbin, 2019; Baluta et al., 2022]). To test the adequacy of calibration the authors compared the trajectories and moments obtained using calibrated model with the data.

Namely, the authors first estimated using OLS the terms of trade shocks ε_t^{TOT} and foreign interest rate shocks ε_t^{prem} based on the historic oil price and foreign interest rate series above. Using two series of shocks, the authors simulated the GDP series based on the model and compared it with the actual observed statistics (Figure 1). It turned out that the GDP series reproduced by the model qualitatively coincides with its actual trends.



Note. The GDP is presented as a % deviation from the trend (HP filter with a parameter value of 1600) for model-simulated series, and as a % deviation from the long-term trend with a constant trend of 1% per year for data.

Fig.1. **The GDP** (Y-Axis, %)**, 2014 Q1 - 2019 Q2**

Second, the authors compared the correlation matrix of the main model variables with the correlations observed in the statistics (Table 2), as well as standard deviations (Table 3). Almost all correlations coincide with respect to their sign (positive or negative), and most of the pairs are close in value.

Table 2

Correlation Matrix of the Main Model Variables, 2014 Q1 to 2019 Q3

Data/Simulations	Real GDP	Households' Real Consumption	Oil Price	MIACR Rate	Inflation
Real GDP	1/1	0.94/0.96	0.81/0.69	–0.44/–0.78	0.16/–0.05
Households' Real Consumption	0.94/0.96	1/1	0.82/0.85	–0.38/–0.86	0.17/–0.14
Oil Price	0.81/0.69	0.82/0.85	1/1	–0.61/–0.75	–0.08/–0.21
MIACR Rate	–0.44/–0.78	–0.38/–0.86	–0.61/–0.75	1/1	0.62/0.44
Inflation	0.16/–0.05	0.17/–0.14	–0.08/–0.21	0.62/0.44	1/1

Notes. 1. The first number in the xx/yy pair corresponds to the data and the second one corresponds to the simulated series. 2. The long-term trend was subtracted from the data (HP filter with a parameter value of 1600).

Table 3

Standard Deviations of the Main Model Variables

	Data, 1999 Q2 to 2019 Q3	Data, 2010 Q1 to 2019 Q3	Data, 2014 Q1 to 2019 Q3	Simulations
Real GDP	1,4%	0,9%	0,9%	0,7%
Households' Real Consumption	1,9%	1,9%	2,0%	1,7%
Oil Price	14,3%	12,9%	14,4%	14,5%
MIACR Rate Deviation from Long-Term Trend	2,4%	1,7%	1,9%	1,1%
Inflation Deviation from Long-Term Trend	1,3%	1,1%	1,3%	0,9%

The results of both tests indicate that the model calibration is adequate.

3. The Negative Impact of the Economy Being at the ZLB

Reaching the ZLB leaves the regulator unable to lower the interest rate further. Inflation is determined by free market forces, which leads to its excessive volatility and harms the economy.

If the monetary authority follows the standard Taylor rule

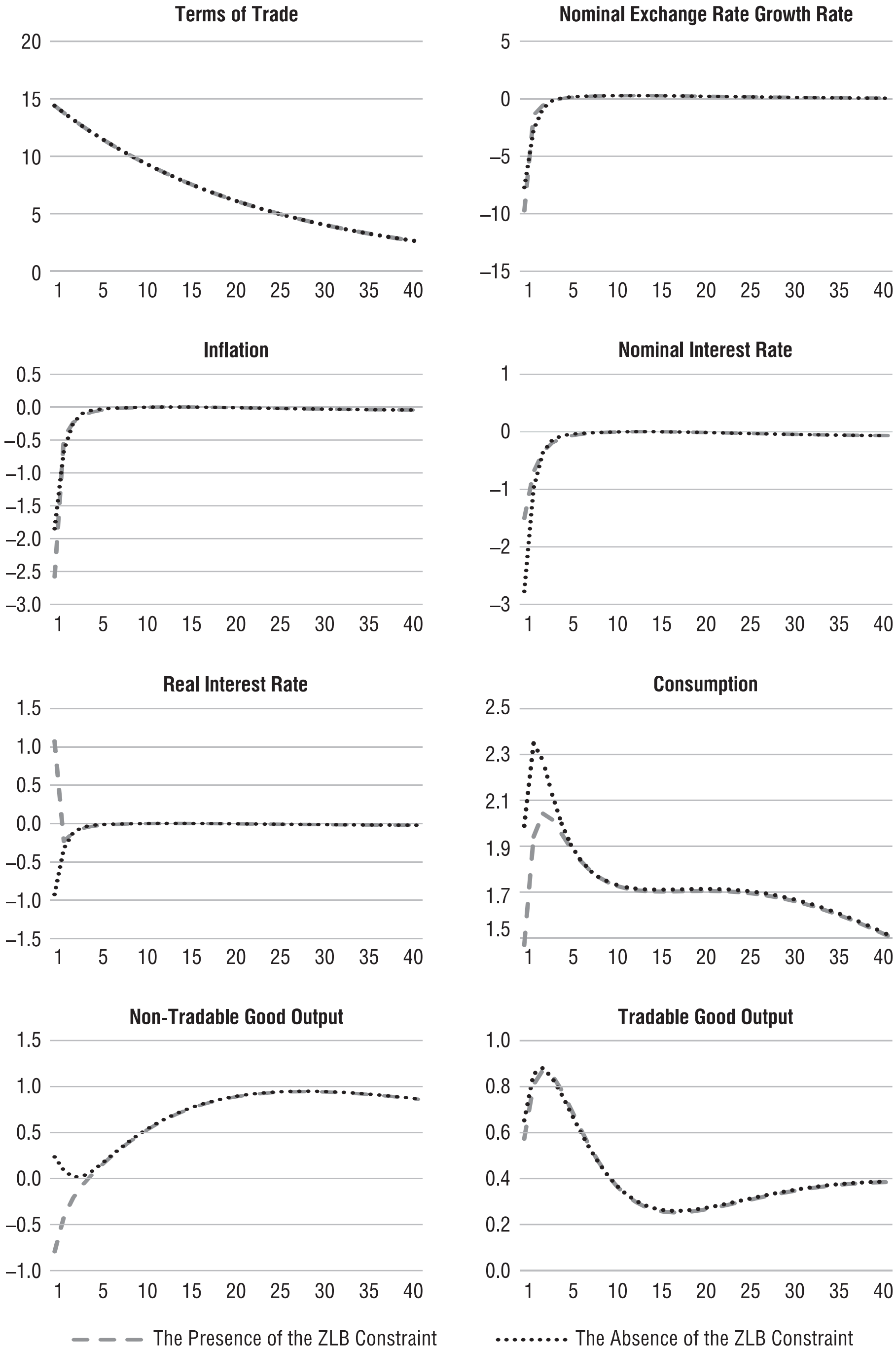
$$R_t^l - R^{l,ss} = 1.5\,(\pi_t - \pi^{ss}),$$

then even the TOT shock of less than 1 standard deviation brings the economy to the ZLB[4] (Figures 2 and 3).

As a result of a positive TOT shock, the exchange rate strengthens. Inflation has an external component determined by the exchange rate. Therefore, inflation falls, which forces the regulator to lower the interest rate. In the demonstrated case, if there is a lower bound on rates, the regulator can reduce the rate only to 0% p.a. from the long-term value of 6% (Figure 2), whereas if there is no lower bound, the rate can be cut below 0%. If there is a limit, inflation falls below what it would be if there were no such limit. The difference between the nominal interest rate and expected inflation leads to a higher real interest rate if there is a lower bound on nominal rates.

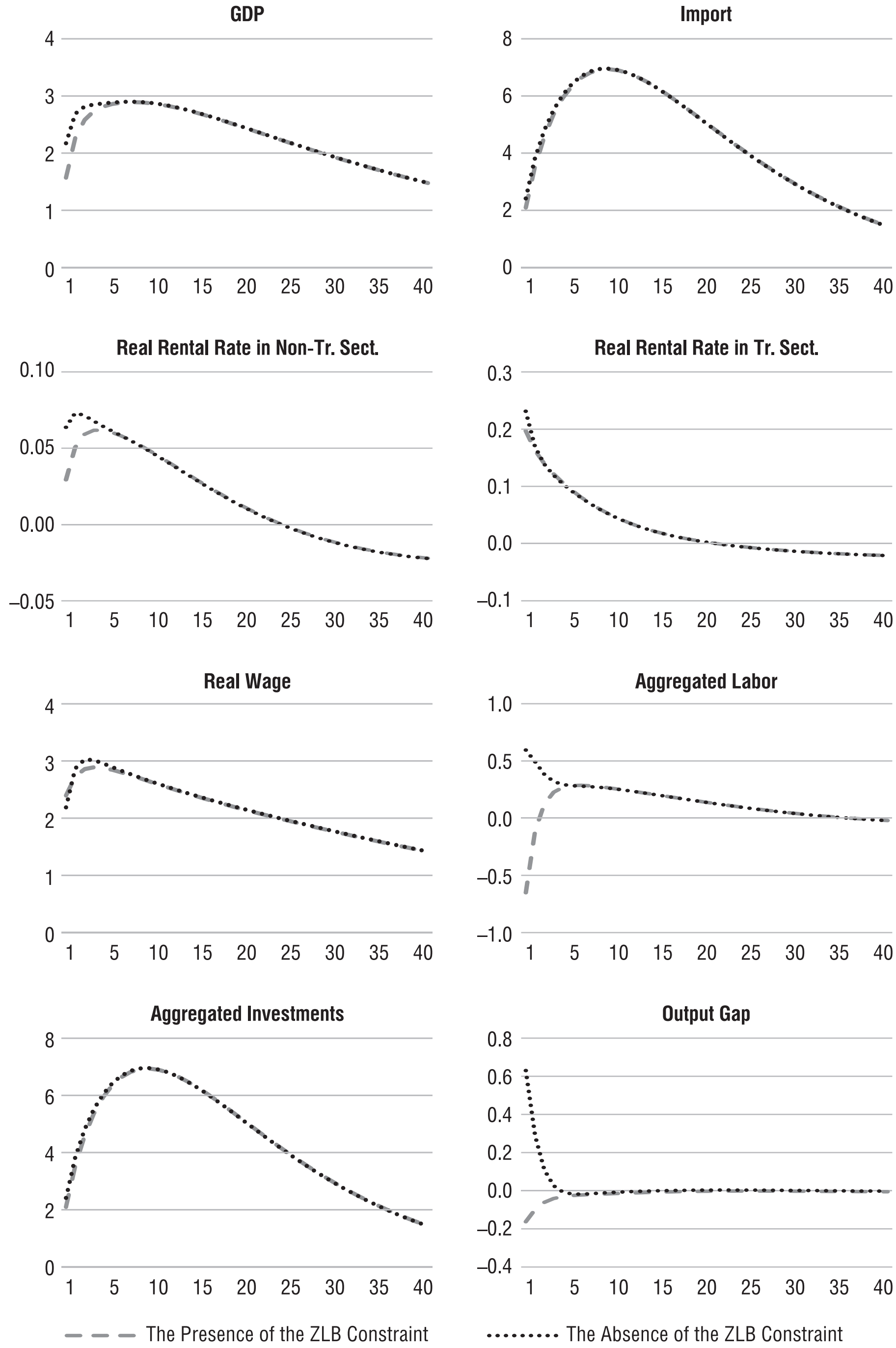
A further difference between the regimes is due to the difference in how the real interest rate responds. A higher real rate in the presence of a nominal interest rate limit leads households to exhibit more saving behavior and to increase consumption to a lesser extent. Lower

[4] The curve "nominal interest rate" for the case "ZLB constraint" (Figure 2) drops from 0 to the lower bound of -0.015. This corresponds to a fall in the rate from 6% to 0% in annual terms. Therefore, the case "ZLB constraint" corresponds to hitting the ZLB at the same period as the shock occurred.



Note. The figure corresponds to the case of the Taylor rule parameters $\rho_{inf} = 1.5$, $\rho_y = 0$, $\rho_r = 0$. Gray curve indicates the presence of the ZLB constraint. Black curve indicates the absence of the ZLB constraint.

Fig. 2. **The Response of Model Variables to a Positive 1-Standard-Deviation TOT Shock** (Y-Axis, % Deviation from Steady State) **and Quarters** (X-Axis)



Note. The figure corresponds to the case of the Taylor rule parameters $\rho_{inf} = 1.5$, $\rho_y = 0$, $\rho_r = 0$. Gray curve indicates the presence of the ZLB constraint. Black curve indicates the absence of the ZLB constraint.

Fig. 3. **The Response of Model Variables to a Positive 1-Standard-Deviation TOT Shock** (Y-Axis, % Deviation from Steady State) **and Quarters** (X-Axis)

consumer demand leads to lower output growth in the non-traded sector, while output in the traded sector changes insignificantly. Aggregate output demonstrates lower growth, which leads to lower growth of both the cost of production factors and production factors as such.

The authors note that the impact of the shock is asymmetric: the above differences between the regimes occur only under a positive shock to terms of trade, while under a negative shock the regimes exhibit the same behavior. This means that both observed average inflation and output are lower under the ZLB condition.

As for volatility, inflation volatility increases when there is an interest rate limit, whereas output volatility and nominal interest rate volatility both decrease. This means that a regulator seeking to reduce inflation volatility alone is negatively impacted by the presence of an interest rate limit. However, if the regulator uses combined criteria, it may benefit from having a limitation. As will be seen later, the regulator can indeed improve the value of the target function to a small extent as a result of an interest rate limit.

Table 4 demonstrates the idea presented earlier that resource-rich economies have slightly less volatility in some key macroeconomic variables under the ZLB constraint. The gain in volatility reduction is greater the higher the probability of the economy being at the ZLB. This gain is relatively small.

Table 4

Standard Deviations of GDP and Consumption Depending on the Presence or Absence of ZLB Constraint

Taylor Rule Parameters	Absence of the ZLB Restriction		Presence of the ZLB Restriction		The Probability of Being at the ZLB (%)
	Consumption	GDP	Consumption	GDP	
$\rho_r = 0.902$, $\rho_{inf} = 1.49$, $\rho_y = 0$ (Estimated TR Parameters Using OLS)	12.56	5.38	12.54	5.36	0.28
$\rho_r = 1$, $\rho_{inf} = 0.82$, $\rho_y = 0$ (One of the Optimal Rules, Table 5)	12.92	5.54	12.71	5.39	8.28

Note. In % deviation from the steady state. TR - Taylor rule.

4. Monetary Authority's Objective

The authors assume that the monetary authority follows the Taylor rule (1). The monetary authority seeks to optimize the loss function. The search for optimal rules takes place in the space of Taylor rule parameters ρ_{inf}, ρ_y, ρ_r.

In a similar fashion to [Adolfson et al., 2011; Eggertsson, 2011; Williams, 2009], the authors used the classical central bank loss function in the form of a weighted sum of inflation variance, output gap, and interest rate:

$$L = Var(\pi_t) + \alpha Var(gap_t) + \beta Var(R_t^l). \quad (6)$$

Minimizing inflation variance $Var(\pi_t)$ is the primary objective of the MP authority, so this term is presented with a coefficient of 1 in the loss function. $Var(gap_t)$ corresponds to the desire of the MP authority to smooth both inflation and output gap. The last summand, the variance of the nominal interest rate $Var(R_t^l)$, is required to exclude the cases where inflation and output gap are smoothed by an excessively aggressive response by the MP authority in accordance with the Taylor rule. Aggressive interest rate changes are not usually observed in reality, as each change imposes a non-financial cost on the MP authority. Among all of the possible variants of the target function, the authors chose four: $Var(\pi_t)$, $Var(\pi_t) + Var(gap_t)$, $Var(\pi_t) + 0.35Var(R_t^l)$, and $Var(\pi_t) + Var(gap_t) + 0.35Var(R_t^l)$. The $\beta = 0.35$ value was chosen, firstly, similarly to [Adolfson et al., 2011], and secondly, with the given value of the coefficient, interest rate variation makes an impact on the loss function comparable to inflation variation.

The authors generated five sequences of terms of trade shocks ε_t^{TOT} and foreign interest rates ε_t^{prem} with 1,000 points each.[5] For each pair of 1,000-point chains, the authors calculated the model solution using the OccBin tool. Moving in the space of parameters ρ_{inf}, ρ_y, ρ_r, the authors found parameter values that minimized criteria of type (6).

5. Optimal Simple MP Rules

The optimal parameter values are presented in Table 5.

The values of the parameters in Table 5 are presented for specification of the Taylor rule

$$R_t^l - R^{l,ss} = \rho_r(R_{t-1}^l - R^{l,ss}) + \rho_{inf}(\pi_t - \pi^{ss}) + \rho_y(GDP_t/GDP_t^{pot} - 1), \quad (7)$$

instead of specification due to the possibility of parameter ρ_r to tend to unity and the need to limit the parameters ρ_{inf}, ρ_y from above. Note that the estimated parameter values for the Taylor rule (Table 1) $\rho_r = 0.902$, $\rho_{inf} = 0.149$ correspond to the value $\rho_{inf} = 0.146$ in the specification.

[5] The need to use several short sequences of shocks instead of a single long one arises, firstly, from a more-than-linear increase in the OccBin tool run time depending on the sequence length and, secondly, from the ability to refine the research results by generating additional iterations.

Table 5 contains information broken down by:

- Two long-term inflation values: 0% p.a. and 4% p. a.
- Four kinds of loss functions (6).
- The presence or absence of a zero lower bound condition (2) is indicated in the "Avail. of ZLB constr." column. If the bound is set, the column t_1 indicates the probability of the economy being at the lower bound of interest rates. Otherwise, the probability of the interest rate being below the zero (unset) lower bound is indicated.

During the optimization process the authors assumed that parameters ρ_{inf}, ρ_y, ρ_r are non-negative, the autocorrelation coefficient ρ_r is lower than or equal to 1, and the degree of response to inflation ρ_{inf} is lower than or equal to 3.5.

As the table shows, the differences between the regimes with and without the ZLB restriction are as follows. Firstly, in the presence of an interest rate limit (2), the probability of being at the rate bound is lower than being below the bound when there is no limit. The reason for this is the increased inflation volatility under the ZLB restriction, which leads to such optimal parameters that the probability of reaching the boundary is reduced. Secondly, for the same reason, the values of the criteria are higher when there is a rate bound. Thirdly, the parameter for policy response to inflation ρ_{inf} decreases when the limit (2) is imposed. This can be explained by the undesirability of reaching the interest rate bound, the probability of which is higher in the event of more aggressive rate changes at high values of the parameter ρ_{inf}. Fourthly, the policy response parameter for output gap ρ_y increases when the limit is imposed on the rate. This result turns out to be unexpected and counter-intuitive given the undesirability of reaching the interest rate bound. It can be explained by the fact that an increase in the parameter ρ_y leads to an increase in inflation volatility and to a decrease in output gap volatility. The decrease in output gap volatility offsets the increase in inflation volatility. Finally, the value of the rate autocorrelation parameter ρ_r also increases when the limit is imposed. The tendency of the rate autocorrelation parameter to reach values above 1 in the optimal rules is also noted in [Adolfson et al., 2014]. In this paper's model, since the decisions of economic agents are based on future inflation, which is reflected in the Phillips curve equation, lower expected future inflation results in lower current inflation. In the case of higher values ρ_r ($\rho_r = 1$, Figure 4) corresponding, all other things being equal, to a longer return

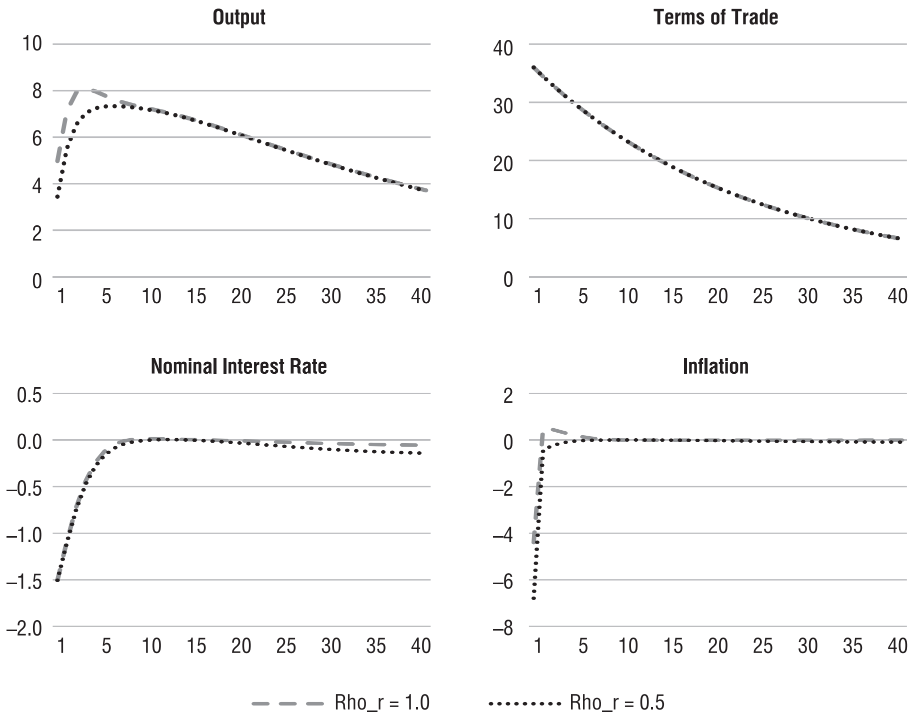
Table 5

Parameters of Optimal Rules

Loss Function / Parameter	Avail. of ZLB Constr.	ρ_{inf}	ρ_y	ρ_r	t_1 (%)	Loss Function Value	Value $Var(\pi_t)$	Value $Var(gap_t)$	Value $Var(R_t^l)$
Long-Term Inflation Rate - 0%									
$L = Var(\pi_t)$	No	3.5	0	0.95	39.78	0.034	0.034	0.517	0.546
	Yes	3.11	0	1	33.08	0.313	0.313	0.163	0.212
$L = Var(\pi_t) + Var(gap_t)$	No	1.29	0	0.47	39.58	0.345	0.196	0.149	0.459
	Yes	1.03	0.04	1	28.5	0.437	0.341	0.096	0.095
$L = Var(\pi_t) + 0.35Var(R_t^l)$	No	1.44	0	1	36.82	0.184	0.093	0.352	0.262
	Yes	0.76	0	1	26.14	0.365	0.342	0.093	0.068
$L = Var(\pi_t) + Var(gap_t) + 0.35Var(R_t^l)$	No	0.48	0	0.86	29.2	0.414	0.246	0.136	0.094
	Yes	0.82	0.01	1	26.74	0.459	0.341	0.092	0.074
Long-Term Inflation Rate - 4%									
$L = Var(\pi_t)$	No	3.5	0	0.95	20.96	0.033	0.033	0.608	0.519
	Yes	3.5	0	1	20.66	0.156	0.156	0.296	0.304
$L = Var(\pi_t) + Var(gap_t)$	No	1.28	0.17	0.51	20.42	0.366	0.224	0.142	0.443
	Yes	1.05	0.25	1	13.28	0.380	0.248	0.132	0.164
$L = Var(\pi_t) + 0.35Var(R_t^l)$	No	1.47	0	1	14.58	0.178	0.089	0.423	0.254
	Yes	0.82	0	1	8.28	0.225	0.186	0.220	0.112
$L = Var(\pi_t) + Var(gap_t) + 0.35Var(R_t^l)$	No	0.45	0.06	0.91	4.52	0.431	0.267	0.135	0.086
	Yes	0.56	0.11	1	5.96	0.421	0.259	0.130	0.090
$L = Var(\pi_t)$, TR Estimated Parameters	No	0.146	0	0.902	0.32	0.403	0.403	0.082	0.028
$L = Var(\pi_t)$, TR Estimated Parameters	Yes	0.146	0	0.902	0.28	0.404	0.404	0.081	0.028

Note. $Var(x_t)$ means variation of the variable x_t.

of the rate to its long-term equilibrium, the regulator promises a longer counteraction to the causes of inflationary change, which reduces the inflation response. The decrease in the interest rate response under higher values of ρ_r is the result of the mutual influence of variables in the equilibrium model, reflecting the knowledge of agents about the intention of the regulator.[6] This explains the optimality of high autocorrelation values for the interest rate ρ_r.



Note. The Taylor rule parameters: ρ_r = 1 for gray curve, ρ_r = 0.5 for black curve. Both curves correspond to ρ_{inf} = 0.82. Nominal interest rate and inflation are given quarterly.

Fig. 4. **The Impulse Response of Certain Model Variables to a Positive 2.5-Standard-Deviation TOT Shock** (Y-Axis, % Deviation from the Steady State) **and Quarters** (X-Axis)

The optimal rules corresponding to different long-term inflation values of 0% and 4% differ in the probability of being at the ZLB and criteria values. At 0% inflation, the probability of staying at the ZLB is higher because the long-term interest rate of 2% p.a. is closer to the ZLB than it would otherwise be at 6% p.a. Accordingly, when inflation is 0% under the ZLB restriction, the value of the loss function is higher because the monetary authority has a decreased ability to respond to fluc-

[6] If equation (7) were considered isolated from the model, then at high values of ρ_r the interest rate response would be higher, and the return to long-term equilibrium would be longer.

tuations in macro variables. The long-term value of inflation does not significantly affect the optimal values of the rule parameters.

The presence of output variation $Var(gap_t)$ in the penalty function (6) causes the interest rate to respond to changes in output gap under the optimal rule. Otherwise, $\rho_y = 0$, and the rate is unresponsive to output gap. Also, the presence of output variation in the penalty function reduces the response to changes in inflation ρ_{inf} and the autocorrelation coefficient ρ_r.

Similarly, the presence of interest rate variation $Var(R_t^l)$ in the penalty function causes the interest rate autocorrelation coefficient ρ_r to increase, while the inflation and output gap response coefficients ρ_{inf}, ρ_r decrease.

The case of the loss function $L = Var(\pi_t) + Var(gap_t) + 0.35Var(R_t^l)$ with a long-term inflation rate of 4% is distinct from the above analysis. The unexpected thing is that, firstly, the value of the loss function declines when the lower bound is introduced and, secondly, the probability of reaching the interest rate bound rises when the lower bound is introduced. It would be expected that the presence of the interest rate bound should restrict monetary policy actions and lead to excessive volatility of variables and an increase in the value of the loss function when reaching the rate bound. Indeed, in the presence of an interest rate bound, inflation volatility increases as the Taylor rule ceases to apply at the rate bound. However, at the interest rate bound, firstly, the nominal interest rate itself stops changing, and, secondly, output gap is less responsive to shocks, which results in a lower interest rate and output gap volatility. The reduction in interest rate and output gap volatility is greater than the increase in inflation volatility, which explains the effect.[7] Overall, Table 5 shows that adding interest rate volatility and/or output gap volatility to the loss function reduces the difference between the loss function values in the presence and absence of an interest rate bound.

6. The Probability of Being at the ZLB and at the Effective Lower Bound (ELB)

The values of probability for reaching the ZLB at 4% inflation under optimal rules (Table 5) are quite high—from 6% in the case of a loss function that includes the volatility for all three variables to 21% in the

[7] For the case from Table 4, in the absence of a ZLB constraint under the parameter values $\rho_{inf} = 0.45$, $\rho_y = 0.06$, $\rho_r = 0.91$ the loss function value is $Var(\pi_t) + Var(gap_t) + 0.35Var(R_t^l) = 0.267 + 0.135 + 0.35 \times 0.086 = 0.431$. Under the ZLB constraint and under the same parameter values the loss function value is $Var(\pi_t) + Var(gap_t) + 0.35Var(R_t^l) = 0.290 + 0.109 + 0.35 \times 0.077 = 0.425 < 0.431$. Further optimization of the parameters, as can be seen from Table 4, reduces the value of the loss function to 0.42 and changes the value of the loss function components.

case of a loss function for inflation variation. A high probability of being at the ZLB suggests that the desire to avoid the zero bound at all costs should not be attributed to a rational monetary policy authority: a high probability of staying at the bound may be the optimal choice. A comparison of the estimated Taylor rule parameter values (Table 1) for Russia with the optimal rules indicates that the actual policy is closer to that of regulating inflation volatility with a reluctance to abruptly change the interest rate, which corresponds to a 6–8% probability of staying at the rate bound in the optimal rules. Estimated TR parameters using OLS (Table 5, last and second to last lines) lead to a 0.3% probability of being at the ZLB. However, a single TOT shock can push the economy to the ZLB only if it exceeds +5.4 standard deviations. The probability of such an event is less than $3*10^{-8}$, while the probability for an interest rate shock is even lower. Therefore, a 0.3% probability of the economy being at the ZLB can only be realized through a prolonged series of major positive terms-of-trade shocks.

Note that the probability of being at the ZLB grows with the fall of the inflation steady state value. Table 6 illustrates this.

Table 6

The Probability of Being at the ZLB, Depending on the Inflation Steady State Value

Inflation Steady State (% p.a.)	Nominal Interest Rate Steady State (% p.a.)	The Probability of Being at the ZLB (%)
4	6	0.3
3	5	0.8
2	4	3.2

Note. The results are given for the case of estimated Taylor rule parameters using OLS.

The zero bound of interest rates is not the only boundary near which monetary policy can be ineffective. For example, the households' saving behavior may qualitatively change if the profitability of placing deposits is much less than investments in other alternative instruments. The problem of not being able to cut the rate below a certain non-zero threshold has been called the effective lower bound (ELB) of interest rates. The trend towards a shift in focus from ZLB to ELB is aptly expressed by Governor of the Central Bank of Norway Øystein Olsen, "Before the financial crisis, most economists regarded zero as the lower bound for policy rates. At the same time, this limit was mainly of theoretical interest. Experience from the past decade has changed perspectives on the lower bound, and authors now talk of 'the effective lower bound' rather than 'the zero lower bound.'"[8]

[8] https://www.norges-bank.no/en/news-events/news-publications/Speeches/2020/2020-10-06-cme/.

In relation to Russia, the instrumental opportunity leading to the emergence of ELB may be an investment in foreign currency. This is due to the fact that, over the past decade, keeping funds in foreign currency has turned out to be de facto highly profitable: the average annual return on holding funds in US dollars from 1999 to 2020 was 5.3%; from 2007 to 2020, 7.6%; from 2013 to 2020, 12.8%. At the same time, the average rate on placing deposits in national currency for a period of one to three years in the period from 2014 to 2020 was 7.2%, having decreased to 4.5% in 2020. Thus, the profitability of keeping funds in foreign currency exceeded the profitability of deposits in national currency. At the same time low interest rates on deposits in foreign currency do not create an incentive for keeping foreign currency in banks.

The authors leave the question of the exact ELB level for Russia and whether this level can be determined for other studies. Here, in Table 7, the authors present the probabilities of the economy hitting the potential lower bounds of rates, going over their values from 0% (ZLB) to 3% with a 0.5% step. The probabilities are given for the case of the estimated parameters of the Taylor rule using OLS (Table 1).

Table 7

The Probability of Being at the Rate Bound, Depending on the Value of the Bound

Lower Bound Value of the Rate (%) p.a.	The Probability of Being at the Rate Bound (%)
0 (ZLB)	0.3
0.5	0.4
1.0	0.8
1.5	1.9
2.0	3.2
2.5	>2.38
3.0	>3.88

Notes. 1. The results are given for the case of estimated Taylor rule parameters using OLS, 4% p.a. inflation steady state and 6% p.a. nominal interest rate steady state. 2. For the 2.5% and 3% rate bound the authors present only the lower estimate of the probability, since they were unable to calculate the equilibrium trajectories of macroeconomic variables with the help of OccBin for some realizations of the sequence of shocks under the estimated TR parameters. The authors can only say that calculation problems arise when the probability of hitting the bound turns out to be high. Therefore, the given estimate is a lower estimate.

As one can see, the probability of hitting the interest rate bound grows rapidly with the growth of the bound value. This means that the Bank of Russia should not ignore the problem of the existence of ELB. At the same time, the optimal policy rules remain qualitatively the same as in Table 3, regardless of the value of the bound.[9]

[9] This is because the optimal rules are weakly dependent on the level of long-term inflation (Table 4). The long-term inflation value is directly connected with the long-term value of the nominal interest rate. Thus, the optimal rules do not change much depending on the long-term level of the interest rate and on the value of the bound.

7. Conclusion

The authors have considered the DSGE model for an open export-oriented economy in which a TOT shock and a foreign interest rate shock can push the domestic nominal interest rate to the zero lower bound. For an export-oriented economy, shocks which drive the economy to the interest rate bound are positive (terms of trade increase, foreign interest rate decrease). Based on the impulse response functions, the authors have shown that the presence of a lower interest rate bound reduces the impact of positive shocks. This is explained by the fact that shocks that drive the economy to the interest rate bound cause inflation to decrease more sharply and the real interest rate to increase. This pushes consumers towards more saving behavior and reduces consumption and output.

In an export-oriented economy inflation volatility increases under the ZLB, while output gap and interest rate volatility decreases. The authors have shown that if the regulator's objective is only to reduce inflation volatility, then the presence of an interest rate limit unambiguously worsens the regulator's target function. If the regulator simultaneously reduces the volatility of several indicators, the negative impact of the limit becomes weaker and, in some cases, may lead to a small gain.

Generally, when the presence of the zero bound negatively affects the regulator's objective, the parameters of the optimal monetary policy rule are such that the probability of the economy being at the zero lower bound is lower than the probability of being below the zero in the case of no boundary. It can be said that the regulator reduces the likelihood of being on the ZLB when it takes into account the ZLB problem. Taking into account the ZLB problem leads to a lesser degree of the regulator's reaction to inflation and higher interest rate persistence. The latter leads to economic agents' expectations that the regulator will counteract the causes of inflation to a greater extent, which implies the agents' expectation of lower inflation and a more moderate inflationary response to shocks.

The authors calibrated the model using the parameters of resource-rich countries. The authors estimated a number of parameters using Russian data, and set long-term inflation at 4% and the interest rate at 6%, which corresponds to the Bank of Russia's target. It was found that the probability of reaching the interest rate bound is only 0.3%. This may be the answer to why exporting economies are rarely seen on the ZLB: the long-term interest rate is too high, even for volatile commodity prices, to push the economy onto the ZLB.

In the optimum, the probability of being at the ZLB is estimated at 6.0–20.1%, depending on the regulator's target. This indicates that the optimal policy does not necessarily need to avoid the ZLB at all costs. According to the OLS estimate of the Taylor rule parameters for Russia, the interest rate reaction to output gap and the interest rate persistence parameter are within the range of values from the optimal rules. At the same time the estimated degree of inflationary response appears to be well below the optimal values.

Due to the fact that the profitability to save money in foreign currency in Russia in recent years is high, the Russian economy may face the ELB problem. The authors estimate that, under the current monetary policy, the likelihood to hit the ELB is significant.

APPENDIXES

Detailed Description of the Model

A complete description of the model used in the paper is provided below.

APPENDIX 1

Households

Each household $i \in [0;1]$ optimizes a utility function reflecting their satisfaction from consumption $C_t(i)$ and their dissatisfaction from labor $L_t(i)$:

$$U(i) = E\sum_{t=1}^{+\infty} \beta^t \left(\ln(C_t(i) - H_t) - \frac{\sigma_L}{1 + l\_el} (L_t(i))^{1+l\_el} \right).$$

The value $H_t = hC_{t-1}$ reflects consumption habits as in [Smets, Wouters, 2007].

Households are assumed to have the monopoly power in the labor market; therefore they set their labor supply based on knowledge of demand for their labor: $L_t(i) = (W_t(i)/W_t)^{-h} L_t$, where $W_t(i)$ and W_t are individual and aggregate nominal wages, and L_t is aggregate labor. In addition, households determine the nominal volume of investments in foreign bonds $Df_t(i)$, as well as the volume of loans provided to other households $Loan_t(i)$. Similar to [Smets, Wouters, 2007], the authors assume that the interest rate on household loans is set by the monetary policy authority. This assumption eliminates the need to define financial intermediaries, such as commercial banks, which transfer to households the interest rate of the monetary policy authority with distortions that are out of the scope of this paper.

Each household in the model owns two types of capital: $K_t^d(i)$ for the needs of the intermediate domestic (non-tradable) product sector and

$K_t^{ex}(i)$ for the needs of the export (tradable) product sector. Two types of capital are not considered to be perfect substitutes. Both types of capital are leased to manufacturers at nominal rental rates ρ_t^d, ρ_t^{ex}. Stocks $K_t^d(i)$, $K_t^{ex}(i)$ are related to the flows of new capital $DK_t^d(i)$, $DK_t^{ex}(i)$, purchased from capital producers at nominal prices Q_t^d, Q_t^{ex} as follows:

$$K_t^d(i) = (1 - \delta_d)\, K_{t-1}^d(i) + DK_t^d(i),$$
$$K_t^{ex}(i) = (1 - \delta_{ex})\, K_{t-1}^{ex}(i) + DK_t^{ex}(i), \tag{8}$$

where δ_d, δ_{ex} are the depreciation rates of capital.

A household's budget constraint is

$$C_t(i) + \frac{Loan_t(i)}{P_t} + \frac{Df_t(i)S_t}{P_t} + \frac{Q_t^d DK_t^d(i)}{P_t} + \frac{Q_t^{ex} DK_t^{ex}(i)}{P_t} =$$
$$= \frac{W_t(i)\mathrm{L}_t(i)}{P_t} + \frac{N_t}{P_t}\, land(i) + R_{t-1}^l \frac{Loan_{t-1}(i)}{P_t} + \frac{\rho_t^d K_{t-1}^d(i)}{P_t} + \frac{\rho_t^{ex} K_{t-1}^{ex}(i)}{P_t} + \tag{9}$$
$$+ R_t^f\, \frac{Df_{t-1}(i)S_t}{P_t} + \frac{\Pi_t^{fiz}}{P_t} + \frac{\Pi_t^d}{P_t} + \frac{\Pi_t^{ex}}{P_t} - \Psi_t^L(W_t(i)/W_{t-1}(i)) - \Psi_t^{Df}(Df_t(i)),$$

where P_t is the final goods price index; S_t is the nominal exchange rate; $land(i)$ is the land owned by the household and rented out to the export good manufacturer; N_t is the rental value of a unit of land; R_t^l is the nominal gross interest rate on loans provided by households to each other as set by the monetary authorities; R_t^f is the nominal gross interest rate on foreign bonds denominated in foreign currency and set by the process (5); Π_t^{fiz}, Π_t^d, Π_t^{ex} are the profits transferred to households by producers of capital and the two types of products; and $\Psi_t^L(W_t(i)/W_{t-1}(i))$, $\Psi_t^{Df}(Df_t(i))$ are costs incurred by a household from changes in nominal wages and investing in non-residential bonds. In accordance with [Rotemberg, 1982], costs are assumed to be increasing quadratic functions of arguments and equal to zero in the long-run equilibrium: $\Psi_t^L = ½w(W_t(i)/W_{t-1}(i) - 1)^2\, W_t L_t/P_t$, $\Psi_t^{Df} = ½d_s(Df_t(i)S_t/P_t Y_t)^2\, Y_t$.

By denoting the Lagrange multiplier under the household budget constrain as $\beta^t \Lambda_t$ and discarding the household index within the symmetric equilibrium analysis (households do not differ from each other), the authors obtain the following necessary optimal conditions for consumption, credit, bond investments, wages, and capital for the intermediate domestic and export product sectors:

$$\Lambda_t = \frac{1}{C_t - hC_{t-1}}, \tag{10}$$

$$\Lambda_t = \beta R_t^l E_t \Lambda_{t+1}/\pi_{t+1}, \tag{11}$$

$$\Lambda_t = \beta R^f E_t \Lambda_{t+1} \frac{S_{t+1}}{\pi_{t+1} S_t} - \Lambda_t \frac{P_t}{S_t} \frac{\partial \Psi_t^{Df}}{\partial Df_t}, \tag{12}$$

$$\sigma_L (L_t)^{l\_el} = \frac{\eta - 1}{\eta} \Lambda_t \frac{W_t}{P_t} - \frac{W_t}{\eta L_t} \Lambda_t \frac{\partial \Psi_t^L}{\partial W_t} - \beta \frac{W_t}{\eta L_t} E_t \Lambda_{t+1} \frac{\partial \Psi_{t+1}^L}{\partial W_t}, \tag{13}$$

$$\begin{aligned} \Lambda_t \frac{Q_t^d}{P_t} &= \beta \Lambda_{t+1} \left(\frac{Q_{t+1}^d}{P_{t+1}} (1-\delta_d) + \frac{\rho_{t+1}^d}{P_{t+1}} \right), \\ \Lambda_t \frac{Q_t^{ex}}{P_t} &= \beta \Lambda_{t+1} \left(\frac{Q_{t+1}^{ex}}{P_{t+1}} (1-\delta_{ex}) + \frac{\rho_{t+1}^{ex}}{P_{t+1}} \right). \end{aligned} \tag{14}$$

In the expressions , $\pi_t = P_t / P_{t-1}$ is the inflation for final goods in national currency.

APPENDIX 2

Manufacturers of Final Goods

Manufacturers of the final goods first buy the differentiated goods $Y_t^d(j)$ in a market under perfect competition from each manufacturer j, $j \in [0;1]$ of the intermediate domestic goods at nominal price $P_t^d(j)$. Manufactures then first aggregate the differentiated goods according to the Dixit-Stiglitz function $Y_t^d = [{}_0\int^1 (Y_t^d(j))^{(\sigma_d - 1)/\sigma_d} dj]^{\sigma_d/(\sigma_d - 1)}$. Second, they aggregate the domestic goods Y_t^d together with the imported goods bought at price $P_t^f Imp_t$ into the final goods Y_t according to the Cobb-Douglas production function

$$Y_t = \frac{(Imp_t)^{\omega} (Y_t^d)^{1-\omega}}{\omega^{\omega} (1-\omega)^{1-\omega}}. \tag{16}$$

Manufacturers sell the final goods at price P_t, aiming to maximize their profit $P_t Y_t - S_t P_t^f Imp_t - {}_0\int^1 P_t^d(j) Y_t^d(j) dj$. The solution to the profit maximization problem under technological constraints provides the following demand function for the product j:

$$Y_t^d(j) = Y_t^d \left(\frac{P_t^d(j)}{P_t^d} \right)^{-\sigma_d}, \tag{17}$$

where P_t^d is the aggregate price of the domestic goods. Then, under the assumption of equilibrium symmetry, the rest of the solution to the problem is

$$\frac{P_t^d}{P_t} Y_t^d = (1-\omega) Y_t, \tag{18}$$

$$\frac{S_t P_t^f}{P_t} Imp_t = \omega Y_t. \tag{19}$$

APPENDIX 3

Intermediate Domestic Goods and Export Goods Manufacturers

Let us assume that manufacturers of intermediate domestic goods operate in a monopolistic competition market. Manufacturers of export goods operate in a perfect competition market. There is a continuum of both types of manufacturers—the authors will number them with the indices $j\in[0;1]$ and $\kappa\in[0;1]$ respectively.

Both types of manufacturers use labor $L_t^d(j)$, $L_t^{ex}(\kappa)$[10], which they hire in the common market at price W_t, and capital $K_{t-1}^d(j)$, $K_{t-1}^{ex}(\kappa)$, which they rent in separate markets from households at nominal rental rates ρ_t^d, ρ_t^{ex} as factors of production during the time period t. Export goods manufacturers, unlike manufacturers of domestic goods, also use a third factor of production, which is land. For the use of land $Land_t(\kappa)$ export manufacturers pay a nominal amount $N_t Land_t(\kappa)$ to households. The specification of the production function for commodity sectors with land as a production factor was also used in the Bank of Canada's ToTEM DSGE model [Murchison, Rennison, 2006]. This assumption takes into account different availability of natural resources: the more land is available to the economy, the lower the cost of extracting a given amount of resources.

The production functions for manufacturers of intermediate domestic and export goods are as follows:

$$Y_t^d(j) = \alpha_d(K_{t-1}^d(j))^{\alpha_d}(L_t^d(j))^{1-\alpha_d}, \tag{20}$$

$$Y_t^{ex}(\kappa) = \alpha_{ex}(K_{t-1}^{ex}(\kappa))^{\alpha_{ex}}(L_t^{ex}(\kappa))^{1-\alpha_{ex}-\gamma_{ex}}(Land_t(\kappa))^{\gamma_{ex}}. \tag{21}$$

Manufacturers decide on the usage of capital $K_{t-1}^d(j)$, $K_{t-1}^{ex}(\kappa)$ during the period t. The backward time shift is due to the assumption that an entrepreneur can only lease to manufacturers the capital that was produced by the end of period $t-1$. This assumption allows us to take into account the capital injection lag.

Domestic intermediate goods manufacturers and export goods manufacturers aim to maximize the expected present value of income

$$E\sum_{t=1}^{+\infty}\beta^t\Lambda_t\frac{\Pi_t^d(j)}{P_t},\ E\sum_{t=1}^{+\infty}\beta^t\Lambda_t\frac{\Pi_t^{ex}(\kappa)}{P_t},$$

where income is determined through the following equations:

$$\begin{aligned}\Pi_t^d(j) = P_t^d(j)Y_t^d(j) - W_t L_t^d(j) - \rho_t^d K_{t-1}^d(j) - \\ - k/2(P_t^d(j)/P_{t-1}^d(j) - 1)^2 P_t^d Y_t^d,\end{aligned} \tag{22}$$

[10] Further, index "*d*" corresponds to a manufacturer of domestic goods, "*ex*" corresponds to a manufacturer of export goods.

$$\Pi_t^{ex}(\kappa) = S_t P_t^{ex} Y_t^{ex}(\kappa) - W_t L_t^{ex}(\kappa) - \rho_t^{ex} K_{t-1}^{ex}(\kappa) - \\ - N_t Land_t(\kappa) - \omega_{ex}/2\ (L_t^{ex}(\kappa)/L_{t-1}^{ex}(\kappa) - 1)^2 W_t L_t^{ex}. \qquad (23)$$

The quadratic terms in the expressions reflect the costs of changing the amount of hired labor $L_t^{ex}(\kappa)$ and nominal price $P_t^d(j)$ in accordance with [Rotemberg, 1982].

The first-order conditions on capital, labor, and land for the export manufacturer problem after switching to aggregate variables are as follows:

$$\rho_t^{ex} K_{t-1}^{ex} = \alpha_{ex} S_t P_t^{ex} Y_t^{ex}, \qquad (24)$$

$$W_t L_t^{ex} + \omega_{ex}\left(\frac{L_t^{ex}}{L_{t-1}^{ex}} - 1\right)\frac{L_t^{ex}}{L_{t-1}^{ex}}\ W_t L_t^{ex} - \\ - \beta\omega_{ex} E_t \frac{\Lambda_{t+1}}{\Lambda_t}\frac{P_t}{P_{t+1}}\left(\frac{L_{t+1}^{ex}}{L_t^{ex}} - 1\right)\frac{L_{t+1}^{ex}}{L_t^{ex}}\ W_{t+1} L_{t+1}^{ex} = \\ = (1 - \alpha_{ex} - \gamma_{ex}) S_t P_t^{ex} Y_t^{ex}, \qquad (25)$$

$$Land_t N_t = \gamma_{ex} S_t P_t^{ex} Y_t^{ex}, \qquad (26)$$

where P_t^{ex} is the external price of the exported goods. The terms of trade are defined as

$$\tilde{P}_t^{res} = P_t^{ex}/P_t^f$$

and are described by the process.

By maximizing the objective function for the domestic intermediate goods manufacturer the authors obtain:

$$\rho_t^d K_{t-1}^d = \alpha_d Mc_t P_t^d Y_t^d, \qquad (27)$$

$$W_t L_t^d = (1 - \alpha_d) Mc_t P_t^d Y_t^d, \qquad (28)$$

$$Mc_t = \frac{\sigma_d - 1}{\sigma_d} + \frac{k}{\sigma_d}\left(\frac{P_t^d}{P_{t-1}^d} - 1\right)\frac{P_t^d}{P_{t-1}^d} - \\ - \beta\frac{k}{\sigma_d} E_t \frac{\Lambda_{t+1}}{\Lambda_t}\frac{Y_{t+1}^d}{Y_t^d}\frac{P_t}{P_{t+1}}\left(\frac{P_{t+1}^d}{P_t^d} - 1\right)\left(\frac{P_{t+1}^d}{P_t^d}\right)^2. \qquad (29)$$

The value of Mc_t is interpreted as the marginal cost of the manufacturer.

APPENDIX 4

Manufacturer of Capital

A manufacturer of capital buys investments Inv_t^d, Inv_t^{ex} in the final goods market to create capital DK_t^d, DK_t^{ex}. The manufacturer sells new capital to households at nominal prices Q_t^d, Q_t^{ex}. The technology for transforming investment into capital takes into account the non-linearity of capital production costs and is described by the following equations:

$$DK_t^d = Inv_t^d - \Psi_t^d(Inv_t^d/Inv_{t-1}^d)Inv_t^d,$$
$$DK_t^{ex} = Inv_t^{ex} - \Psi_t^{ex}(Inv_t^{ex}/Inv_{t-1}^{ex})Inv_t^{ex}, \quad (30)$$

where

$$\Psi_t^d(Inv_t^d/Inv_{t-1}^d) = ½k_d\,(Inv_t^d/Inv_{t-1}^d - 1)^2,$$
$$\Psi_t^{ex}(Inv_t^{ex}/Inv_{t-1}^{ex}) = ½k_{ex}\,(Inv_t^{ex}/Inv_{t-1}^{ex} - 1)^2.$$

The goal of the capital manufacturer is to maximize expected present value of income

$$E\sum_{t=1}^{+\infty}\beta^t\Lambda_t\frac{\Pi_t^{fiz}}{P_t}.$$

The expression for current period profit Π_t^{fiz} is

$$\Pi_t^{fiz} = Q_t^d DK_t^d + Q_t^{ex}DK_t^{ex} - P_t Inv_t^d - P_t Inv_t^{ex}. \quad (31)$$

Maximization gives the following ratios:

$$\frac{Q_t^d}{P_t}\left(1 - \Psi_t^d - Inv_t^d\frac{\partial\Psi_t^d}{\partial Inv_t^d}\right) = 1 + \beta E_t Inv_{t+1}^d\frac{\Lambda_{t+1}}{\Lambda_t}\frac{Q_{t+1}^d}{P_{t+1}}\frac{\partial\Psi_{t+1}^d}{\partial Inv_t^d}, \quad (32)$$

$$\frac{Q_t^{ex}}{P_t}\left(1 - \Psi_t^{ex} - Inv_t^{ex}\frac{\partial\Psi_t^{ex}}{\partial Inv_t^{ex}}\right) = 1 + \beta E_t Inv_{t+1}^{ex}\frac{\Lambda_{t+1}}{\Lambda_t}\frac{Q_{t+1}^{ex}}{P_{t+1}}\frac{\partial\Psi_{t+1}^{ex}}{\partial Inv_t^{ex}}. \quad (33)$$

If $k_d = k_{ex} = 0$, then the investments are identified with the new capital, and the prices for the capital Q_t^d, Q_t^{ex} are equal to the price of the final goods P_t.

APPENDIX 5

Market Equilibrium

The equilibrium conditions for the labor market, land market, credit market, final goods market, and foreign exchange market are as follows:

$$L_t = L_t^d + L_t^{ex}, \quad (34)$$

$$\int_0^1 Land_t(k)dk = \int_0^1 land(i)di = land, \quad (35)$$

$$\int_0^1 Loan_t(i)di = 0, \quad (36)$$

$$C_t + Inv_t^d + Inv_t^{ex} + \Psi_t^L(W_t/W_{t-1}) + k/2(P_t^d/P_{t-1}^d - 1)^2 P_t^d Y_t^d/P_t + \\ + w_{ex}/2(L_t^{ex}/L_{t-1}^{ex} - 1)^2 W_t L_t^{ex}/P_t = Y_t, \quad (37)$$

$$P_t^{ex} Y_t^{ex} + R_{t-1}^f Df_{t-1} = P_t^f Imp_t + Df_t + P_t^f \Psi_t^{Df}/S_t. \quad (38)$$

Equation (35) means that the supply of land is fixed because the amount of land owned by individual households is assumed to be constant. The last three terms in the left part of (37) are the costs associated with changes in nominal wages, manufacturer prices, and hours hired in the export production sector.

APPENDIX 6

Monetary Policy

The monetary authority is assumed to follow the Taylor rule (1) as long as the nominal interest rate is strictly above zero (2). The MP authority ceases to follow the Taylor rule (1) if the nominal interest rate rests at the zero lower bound (3).

APPENDIX 7

Model Equations

The final model is represented by ratios (4), (5), (8)–(16), (18)–(38), and—depending on whether economy is at the lower bound of the nominal interest rate—expressions (1) or (3).

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